WOLFSON UNIT
FOR MARINE TECHNOLOGY &
INDUSTRIAL AERODYNAMICS

THE RESPONSE OF A TUBULAR BEAM
TO AN INCLINED SLAM IMPACT

Supported by The Department of Energy
through The Offshore Energy Technology Board

Report No 488/1 November 1983

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Wolfson Unit for Marine Technology Report No. 488/1
Technology Reports Centre No OT-0-8357

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FOREWORD

In 1975, with increasing numbers of structures operating in the UK offshore waters, there was renewed interest in improving the estimation of stresses associated with slam loading. The Department of Energy therefore supported, through the Offshore Structures Fluid Loading Advisory Group and the Offshore Energy Technology Board, a series of investigations into slam loading which were conducted by Canham (1977) at the Admiralty Marine Technology Establishment, Holmes et al (1976) Liverpool University, Miller (1977) at The National Maritime Institute and Campbell et al (1977) at The Wolfson Unit, Southampton University. Following this initial work, the Wolfson Unit extended their experimental study into slam loading, as reported by Campbell and Weynberg (1979) and (1980), and The National Maritime Institute reviewed work on slamming and studied the problem of predicting the response of a structure to wave impact, as reported by Miller (1980). In 1981 The National Maritime Institute and The Wolfson Unit were commissioned by The Department of Energy to work on a revision to their Guidance Notes using the understanding gained from the recent work. A working committee was formed to assist with the preparation of draft Guidance Notes and the members were:

Mr S J Burnett MaTSU
Mr I M C Campbell Wolfson Unit MTIA, Southampton
Mr M Cooper Lloyds Register of Shipping
Mr G Lawrence MaTSU (successor to Mr Burnett)
Mr R Mcintosh Department of Energy, PED 5
Mr J Mercier Conoco
Dr D L Miller Global Maritime (formerly with NMI)
Mr J Petri Department of Energy, PED 5
Mr A G Reynolds BP
Mr J A Ridley NMI
Mr P Shorrock CJB Earl and Wright
Mr E J Smit Shell
Mr P H J Verbeek Shell

Co-opted members:

Mr S Leverette Gulf Oil Corporation
Mr R Rowan Roxburgh and Partners

The assistance from this Committee is gratefully acknowledged.
The draft Guidance Notes are presented in an appendix to the report by Ridley (1982) on A Study of Some Theoretical Aspects of Slamming. No. OT-R-82113. During the course of their preparation various aspects of the slamming problem came under close scrutiny and this led to revisions and extensions to the original work by The National Maritime Institute and The Wolfson Unit. This report describes aspects of slam loading and the resulting structural response studied by The Wolfson Unit subsequent to the work described by Campbell and Weynberg (1980) on Measurement of Parameters Affecting Slamming. No. OT-R-3042. It is intended that this report should also complement that by Ridley (1982) previously mentioned.
NOMENCLATURE

$A_i, B_i...F_i$

Constants

$C_{s}(t) = \frac{F_s}{\frac{1}{2} \rho V^2 LD}$

Slam force coefficient as a function of time

$C_{s0}$

Peak slam coefficient ($t=0$)

$D$

Cylinder diameter

$E$

Young's Modulus

$f_i$

Natural frequency in mode $i$

$F$

Force

$F(x,t)$

Force as a function of length and time

$F_s(t)$

Slam force as a function of time

$i$

Mode number

$l$

2nd moment of area

$j$

Integer number

$k_i$

Frequency parameter for mode $i$

$K_d$

Dynamic magnification factor - the ratio of the peak dynamic stress to the corresponding static stress caused by a uniformly distributed peak slam load

$K_m = \frac{M}{F_s L}$

Bending moment coefficient

$L$

Length of tubular beam

$m$

Mass per unit length

$M$

Bending moment

$n$

Forcing frequency

$N_F$

Froude Number

$t$

Time

$v$

Speed of travel of slam along a beam

$V$

Slam velocity

$x,y,z$

Rectangular cartesian co-ordinates

$y_i(x)$

Bending mode shape of a beam as a function of length

$Y_i(t)$

Response as a function of time

$\omega$

Forcing frequency

$\Delta$

Small increment

$\theta$

Angle of inclined slam impact
1. INTRODUCTION

Campbell and Weynberg (1980) in Report No. OT-R-8042 described experiments on the slamming of cylinders. The greatest inconsistencies in this work were between theoretical and experimental results for bending strain distributions and associated dynamic magnification factors. The slam load histories given in Report No. OT-R-8042 have been used as the basis for draft Guidance Notes for the Department of Energy. Part of the work conducted whilst preparing these Guidance Notes was aimed at resolving the aforementioned inconsistencies by improved analysis and the results are described in this report.

In addition, previous work on slam loading on cylinders is reviewed.
2. REVIEW OF SLAM LOADING ON CYLINDERS

2.1 Historical Context

Recent research has revealed a large body of literature on the subject of slamming and extensive bibliographies are given by Holmes et al (1976) and Miller (1980). The work has been instigated to provide data on various water impact problems including: seaplane, helicopter and spacecraft ditching; ship forefoot and flare slamming; planing craft and surface effect craft hull slamming; and wave impact on coastal and offshore structures. However, partly because the geometry of the impacting surfaces has a dominant effect on the slam load history, little has been done to correlate all of this information and where attempts have been made for specific shapes, differences have been found between the results from various theories and experimental data. This conclusion may be found in the notable reviews of slamming by Milwitzky (1952), Arlott et al (1958), Chu and Abramson (1961), Moran (1965) and the International Ship Structures Congress (1976) and possible reasons for the difficulty in correlating results will be discussed later. However, it has been concluded that the experimental basis for the draft Guidance Notes largely overcame the problems of earlier work and produced both a reliable expression for the slam loading on cylinders and an understanding of the consequential dynamic response of simple structures. In order to investigate slam loading from wave impact on offshore structures the main problem was to describe the slam velocity and associated impact angle in statistical terms so as to reflect accurately their occurrence in a given sea state. This problem has been addressed by Ridley (1980) in Report No. OT-R-82113.

2.2 Experimental Determination of Slam Loading

The literature contains descriptions of numerous slam experiments on bodies of various shapes. Some details of tests on cylinders are listed in Table 1 and it is clear from this that there is lack of agreement even amongst recent investigators, although in the discussion of the RINA paper by Miller (1977) there was some consensus of opinion between experimenters that differences in results could be caused by the effect of a non-linear decay in the slam load on inclined impact predictions and hydroelastic
effects. More general comments were made in 1976 by the International Ship Structures Congress Committee II.3, who concluded that "the magnitudes of impact pressure obtained from idealized conditions such as two-dimensional drop tests were much higher than those in seakeeping model experiments", and further work was required to improve correlation. It is also reported that the 1980 American Ship Structures Committee recommended further investigations into slamming. Clearly then it is not straightforward to derive slam loads from experiments and difficulties arise because:

i) Slam loads are transient and in the case of two dimensional impact of a cylinder, rise suddenly. This excites a vibration in the model or test rig which makes measurement of the load or pressure history difficult since they are masked by the response of the transducers. Thus special data analysis techniques are required.

ii) If force transducers are used it is essential to have a stiff test rig so that as many vibration cycles as possible occur during the decay period of the slam load.

iii) Accurate alignment of the test model is necessary if two dimensional impact is to be achieved. Miller (1977) found for his tests in waves that 1° inclination of the cylinder axis was significant and Campbell (1980) found 5° of misalignment affected the transducer responses. However, if experiments do result in three dimensional impact the results may only be interpreted through measurement of the inclination and by using theory to relate the results to the two dimensional case.

iv) Model tests in waves usually result in low Froude numbers at which the slam load is only of a similar magnitude to the other loading and in particular the buoyancy force. Furthermore the waves repeatedly wet the model which affect seriously the pressure transducer responses.

v) Evidence from both theory and experiment suggests that the pressure decays rapidly away from the spray root so care must be exercised when interpreting results from pressure transducers since significant pressure distributions can exist over the diaphragm. The size of the transducer and its location therefore affect the rise time and value of the peak response thus making comparisons between different experiments difficult.
2.2 Theoretical Considerations

Slam loading of engineering structures is a hydroelastic problem and the two dimensional slam load history is just one of the inputs required to yield a solution for the structural response. The various components of a rigorous structural analysis may be summarised as:

i) An initial stressed state exists arising from the gravitational loading on structure and residual stresses.

ii) Superimposed is the transient response from either a two dimensional or three dimensional slam load history and the resulting bulk stresses will be modified near joints or cut outs by stress concentrations.

iii) Subsequent loading from buoyancy, viscous drag and other wave inertia loads will also produce a response.

iv) Variable impact speeds, associated with the structural response, affect the damping and frequency of the response.

v) Two dimensional data on slam loads must be extended to the three dimensional case to take account of real sea conditions.

vi) Mathematical modelling of the structure is required to yield a transient dynamic response from the slam loading.

There are numerous examples in the literature of theoretical treatment of these steps starting with the conventional formulation of the physics of a slam as an inertia loading problem whereby the slam load is derived from the change in momentum of the fluid. Most of such theories cited in the literature use the added mass concept with linearised potential flow theory as first applied to the problem by von Karman (1920) and Wagner (1931). Results specifically applicable to cylinders have been produced by Schnitzer and Hathaway (1953), Fabula (1957) and Fabula and Ruggles (1955), and more recently reworked by Wellcome (1977), Miller (1977), Sarpkaya (1978) and Faltinsen et al (1977). The earlier work has been reviewed from a mathematical standpoint by Moran (1965) who pointed to sources of error in all the conventional solutions and since their effect is difficult to quantify, it is therefore unfortunate that the results of these theories differ widely, not only in the value of the initial slam...
coefficient, but also for the subsequent decay. For example, 'Wagner' type wetting corrections for the rising water surface usually double the value of the initial slam coefficient to 2π. Now since the highest pressures occur at the spray root, it is clear that any slam theory must accurately predict the spray root rise if the associated impact load is to be considered reliable. Experimental data showed that the initial spray root rise could be reliably measured from pressure transducers and was slightly greater than that predicted by the theories of Wagner (1931) or Fabular (1957).

Further defects of the conventional slam theories are their failure to account for water compressibility and air cushioning during the early stages of impact. Although Moran (1965) has also reviewed theoretical work on these topics, they are however seldom referred to by subsequent experimenters who have investigated slamming on cylinders, and yet slight air pressures have been measured by both Arhan (1978) and Campbell (1977). The water compressibility is also affected by aeration. Furthermore, during later stages of impact, the effect of cavitation is ignored by conventional theories, although Moran (1965) considered it should be taken into account in more rigorous numerical solutions, and again Campbell (1980) found possible evidence of this in the form of bubble shedding.

The other components of loading acting on the cylinder vary with immersion and depend on the particular entry conditions. Miller (1977) has summarised the loading regimes for wave impact using the Morrison equation to describe the fully immersed stages of impact. The coefficients are usually experimentally determined. The buoyancy load can be significant and in several experiments it has been subtracted from the resultant measured load in order to obtain the peak slamming and drag loads. The rise of the buoyancy load is often associated with the penetration of static water level although the presence of spray root rise, sheet spray, and cavitation or ventilation could cast doubt on this assumption. Unfortunately, 'conventional' slam theories offer no guidance on this point, as most assume zero gravity and neglect cavitation. Ridley (1982) has shown that the addition of the slam load history used in this work, to the static increase in the buoyancy closely fits the total load histories by other experimenters for the later stages of slamming.
The vibrations of a structure excited by a slam result in small velocity fluctuations which feed back to the hydrodynamics and affect the loading. Moran (1965) points out that both von Karman and Wagner noted that variable entry speed effects could be calculated within the approximations of their theories and continues to consider a free fall slam. Using the same approach Campbell (1980) showed that for the case of a vibrating impact, terms emerged from the forcing side of the equation of motion that were in effect damping and mass, and would hence affect the response of the structure.

Of rather greater significance to the problem of predicting bending stresses in tubular beams is that for many cases of practical interest, it appears that the peak of the response in bending to slam loading occurs well before the corresponding peak response to buoyancy loading. This point is illustrated for quasistatic responses by calculated results given by Campbell and Weynberg (1980) and for dynamic responses by calculated results given by Ridley (1982). Subsequent oscillations from the slam could still add to the responses to other wave loadings dependent on the phase of the responses, the decay in the slam response and the degree to which the slam response is dynamic rather than quasistatic.
3. DYNAMIC RESPONSE OF A STRUCTURE TO SLAM LOADING

3.1 Introduction

Once having obtained an estimate of the slam loading, which for this work is:

\[ \frac{1}{p} \frac{gV^2}{D} \left( 5.15/(1 + 19v_D) + 0.55v_D \right) \text{unit length} \quad \text{eqn (1)} \]

it is necessary to compute the structural response. Various methods have been used by investigators and they fall broadly into two types:

a) Solution of the equation of motion of an analogue of the structure represented by lumped mass and discrete stiffness elements with either a single degree of freedom, used by Miller (1977) and Sarpkaya (1978) or a multi degree of freedom used by Arhan (1978) and in the previous report by Campbell and Weynberg (1980).

b) Solution of the equation of motion of an elastic beam as used by Faltinsen (1977), and as presented herewithin.

The single degree of freedom lumped mass equation of motion can be solved by direct numerical integration using a time step and predictor method to obtain the motions. However for solution of the beam response the two dimensional and inclined impact slam load must either be transformed using harmonic analysis into a suitable form for integration, or an exponential expression for slam load history must be sought. Ridley (1982) adopted the second approach whilst the first is considered in this report. Results from the two approaches have been found to be similar.

3.2 Damping of the Response

Effects of damping can be readily studied using the lumped mass method but it is rather more involved to obtain a damped solution for the beam response. The damping like terms in the equation of motion for a vibrating impact, predicted in the previous report by Campbell (1980), have been found to have only a small effect on the peak response and this form of damping decays with the slam load.
Some estimate of the subsequent damping is required to obtain the decay in the response for fatigue analysis. However, it is very difficult to predict the damping forces in waves because they comprise of viscous structural and hydroelastic components and when the response continues beyond the slam period, the flow regime producing the damping alters.

The structural responses measured during the previous experimental investigation have been studied in an attempt to determine the damping. From a limited number of records of responses with \(V/\sigma = 0.235\) to 0.307 the damping during the slam period was light at between 0.6 and 1.8\% critical. This corresponded to an amplitude ratio between successive cycles of \(R = 0.96\) to 0.89.

3.3 Euler Bernoulli Theory of Flexural Vibration of Beams

This classical solution to the problem of flexural vibration of beams is now applied to the case of a beam supported at both ends and subjected to an inclined slam impact.

\[
M = -EI\frac{\partial^2 y}{\partial x^2} \quad \text{eqn (2)}
\]

and is used in the derivation of the equation of motion:

\[
EI\frac{\partial^4 y}{\partial x^4} + m\frac{\partial^2 y}{\partial t^2} = F(x,t) \quad \text{eqn (3)}
\]

where \(y\) is the deflection of the beam
\(z\) is the longitudinal axis
\(t\) is time
\(EI\) is the stiffness of the beam
\(m\) is the unit mass of the beam
\(F(x,t)\) is the applied loading as a function of position and time

It should be noted that no damping term has been included since this can upset the normal mode relationship of the solution whereby the response in each mode of vibration is independent of other modes.
The solution to the equation of motion is of the form:

$$y(x,t) = \sum_{i=1}^{\infty} Y_i(t)y_i(x)$$  \hspace{1cm} \text{eqn (4)}

There are an infinite number of degrees of freedom each with a different mode shape $y_i$ and associated time response function $Y_i$. The mode shapes may be determined from the solution to the free vibration of the beam i.e. $F(x,t) = 0$ but the response function depends on both the natural frequencies of the beam, $f_i$, and the nature of the forcing function.

The free vibration solution for various end conditions of the beam is given in standard text books e.g. Bishop and Johnson (1970). However, in order to obtain an algebraic solution to the forced vibration problem, the loading $F(x,t)$ must be expressed in a form amenable for integration. The problem may be simplified in some cases by choosing an approximate mode shape and finding approximate solutions to the equation of motion. One suitable method uses the theorem of virtual work whereby the response function $Y_i$ is found from equating the change in strain energy due to a virtual displacement to the work done by inertia and external forces.

3.4 Free Vibration of a Beam

Standard text books give the general form of the functions in the solution of the equation of motion as:

mode shape $y_i(x) = A_i\cos(k_i x) + B_i\sin(k_i x) + C_i\cosh(k_i x) + D_i\sinh(k_i x)$  \hspace{1cm} \text{eqn (5)}

response $Y_i(t) = E_i\sin(2\pi f_i t) + F_i\cos(2\pi f_i t)$  \hspace{1cm} \text{eqn (6)}

where

$$k_i' = \frac{(2\pi f_i)^2}{E} \frac{1}{m}$$  \hspace{1cm} \text{eqn (7)}

and $A_i, B_i, C_i, D_i$ are constants which depend on boundary condition and $E$ and $F_i$ are constants which depend on initial conditions
for a simply supported beam

\[ kL = i\pi \quad Y_i = \frac{E_1 \sin \frac{pi x}{L}}{1} \]

for a built in beam

\[ \cos kL \cosh kL = 0 \quad \therefore i = 1 \quad (kL) = 22.4 \]
\[ \quad \quad 2 \quad 61.7 \]
\[ \quad \quad 3 \quad 120.9 \]

\[ Y_i = A_i \left[ (\cos k_i x - \cosh k_i x) - \frac{(\cos k_i L - \cosh k_i L)(\sin k_i x - \sinh k_i x)}{(\sin k_i L - \sinh k_i L)} \right] \]

An approximate mode shape can be used which satisfies all boundary conditions except the shear force and results in \( \frac{\partial^3 Y}{\partial x^3} = 0 \) at the ends,

this has the form \( y_i = A_i (1 - \cos 2\pi x) \frac{x}{L} \)

Raleigh's method gives the associated frequency parameter as

\[ kL = i\pi \left(\frac{16}{3}\right)^{\frac{1}{4}} \quad i = 1 \quad (kL)^2 = 22.8 \]
\[ 2 \quad 91.2 \]
\[ 3 \quad 205.1 \]

Comparison with the exact solution indicates that a good match for the response is only given in the fundamental mode, \( i = 1 \).

3.5 Theorem of Virtual Work

The response function \( Y_i \) can be found by equating the change in strain energy, due to a virtual displacement, to the work done by inertia and external forces. If there are no external forces this equation reduces to the Raleigh method for finding the natural frequency.

for the solution \( y = \sum_{i=1}^{\infty} Y_i(t) \{x\} \)

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the strain energy in a beam is \( SE = \frac{EI}{2} \int_0^L \left( \frac{\partial^2 y}{\partial x^2} \right)^2 dx \)

and a virtual displacement in mode 1 is \( \Delta y = \Delta Y_1 y_1 \)

change in strain energy = \( \Delta SE = \frac{3SE\Delta y}{3q} = EI \int_0^L \left( \frac{\partial^2 y}{\partial x^2} \right)^2 dx \)

work done by inertia force = \(-m \int_0^L \left( \frac{\partial^2 y}{\partial t^2} \right) dx \Delta y = -m \int_0^L \frac{\partial y}{\partial t} \Delta y \int_0^L \frac{\partial y}{\partial t} dx \)

work done by external force = \( \int_0^L \int_0^L F(x,t) dx \Delta y = \int_0^L \int_0^L F(x,t) y_1 dx \)

Thus

\[
Y_1 + Y_1 EI \int_0^L \left( \frac{\partial^2 y}{\partial x^2} \right)^2 dx = \int_0^L F(x,t) y_1 dx \]

\[
\frac{m \int_0^L \frac{\partial y}{\partial t} \Delta y \int_0^L \frac{\partial y}{\partial t} dx}{m \int_0^L \int_0^L \frac{\partial y}{\partial t} dx} \]

eqn (8)

For free vibration where \( F(x,t) = 0 \) the solution to eqn (8) is

\[ Y_1 = E \sin(2\pi f_1 t) + F_1 \cos(2\pi f_1 t) \]

where \( E \) and \( F \) are constants, \( f_1 \) is the natural frequency in the 1th mode and

\[
f_1 = \frac{EI}{2mm} \int_0^L \left( \frac{\partial^2 y}{\partial x^2} \right)^2 dx \]

\[
\frac{L \int_0^L \frac{\partial y}{\partial t} \Delta y \int_0^L \frac{\partial y}{\partial t} dx}{m \int_0^L \int_0^L \frac{\partial y}{\partial t} dx} \]

eqn (9)

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3.6 Harmonic Analysis of Slam Loading

The slam load could either be described as: i) a series of point loads along the beam which would rise and decay in accordance with the two dimensional slam load history, and this approach was followed in the strip theory given in the previous report by Campbell (1980) and used to obtain expressions for inclined impact slam load; or ii) as a series of point loads each of which is constant as it travels along the beam at the speed with which the spray root moves.

The latter description allows an algebraic solution of the equation of motion by following the method given by Inglis (1951), for the response of a bridge to a moving load, i.e:

A point load on the beam is first represented by a Fourier series and the speed at which it moves is thus represented as a harmonic forcing frequency. The equation of motion is solved for a single load and the response is reverted to that for free vibration when the load leaves the beam. The decaying slam load is represented by a series of travelling point loads and the response to the slam loading is found by summing the response to each point load.

Considering a travelling point load as below:

The velocity of the travelling load \( v \) is found from strip theory for a given slam velocity \( V \) and inclination \( \theta \), as \( V/\tan \theta \). It is convenient to express the load in terms of a half range Fourier series.
The loading in a sine series may be found

\[ \text{as: } \frac{2F}{L} \sum_{j=1}^{\infty} \sin(2j\pi x) \sin\left(\frac{j\pi x}{L}\right) \quad \text{eqn (10)} \]

where \( \omega = \frac{V}{2L} \)

or in a cosine series

\[ \text{as: } \frac{F}{L} + \frac{2F}{L} \sum_{j=1}^{\infty} \cos(j\pi x) \cos\left(\frac{j\pi x}{L}\right) \quad \text{eqn (11)} \]

where \( n = \frac{V}{L} \)

3.7 **Forced Vibration of a Beam with Slam Loading**

3.7.1 For a Simply Supported Beam:

The equation of motion may be integrated directly using the harmonic analysis of the slam load. For a single travelling load the equation of motion becomes:

\[ E I \frac{d^4 y}{dx^4} + m \frac{d^2 y}{dt^2} = \frac{2F}{L} \sum_{j=1}^{\infty} \sin(j2\pi x) \sin\left(\frac{j\pi x}{L}\right) \quad \text{eqn (3)} \]

Standard text books show how this may be integrated directly.

The complete solution includes the natural response and with the initial conditions of \( y = \dot{y} = 0 \) at \( t = 0 \) gives:

\[ y = \frac{2FL^3}{k^4EI} \sum_{i=1}^{\infty} \sin\left(\frac{i\pi x}{L}\right) \left( \sin(i2\pi \dot{t}) - \frac{i\omega}{2\pi} \sin(i^2 \omega \Pi t) \right) \quad \text{eqn (12)} \]

\[ \frac{i^4 - i^2 \omega^2}{k^2} \]

It is of interest to note that for this single travelling load a maximum occurs in the first mode response when the frequency of the travelling load equals the natural frequency for the first mode. This implies that there is an inclination at which the beam response would be greatest; however the reduction in slam force with inclination masks this response when the complete slam load is considered.
In higher modes the maximum response occurs when the frequency of the travelling load \( \omega = 1f \); however the natural frequency of the higher modes is \( 1^2f \).

When the load reaches the end of the beam the response reverts to that for free vibration with the displacements and velocities of the two solutions matched, giving:

\[
y = \frac{2FL^3}{\pi^4EI} \sum_{l=1}^{\infty} \frac{\sin \left( \frac{\pi X}{L} \right)}{l^4} \left[ \frac{(\cos \pi \cos \frac{2\pi f}{\omega} - 1) \sin^2 \frac{2\pi f t}{\omega} - (\cos \pi \sin \frac{2\pi f}{\omega} \cos \frac{2\pi f t}{\omega})}{1 + \frac{1^2 \omega^2}{f^2}} \right]
\]

Eqn (13)

The bending moment can be obtained from eqn (2)

\[
M = E I \frac{\partial^2 y}{\partial x^2} = \frac{2FL}{\pi^2} \sum_{l=1}^{\infty} \frac{l^2 \sin \left( \frac{\pi X}{L} \right)}{l} Y_l
\]

where \( Y_l \) is now the response terms in square brackets in eqn (11). A check of this solution can be obtained by comparing the result for a very slow moving load (\( \omega \ll p \)) with that from statics.

\[
Y_l \approx \frac{\sin 2\pi \omega t}{t^4}
\]

\( Y_l \) is therefore a maximum when \( \omega t = 1 = \frac{1v}{2L} \) and \( \omega t \) was defined as \( \omega t = \frac{v}{2L} \).

So for the first mode the maximum bending moment occurs when \( a = \frac{1}{2} \) the load is at the centre of the beam at which time \( Y_1 = 1 \) and \( M = 0.203FL \) whereas from statics \( M = 0.250FL \). This indicates the dominance of the first mode in the response.

3.7.2 For a Built-In Beam:

The equation of motion may be solved using the theorem of virtual work.

Putting the approximate mode shape and cosine series into eqn (8) from the theorem of virtual work gives:
\[
\ddot{y}_i + \frac{y_i f^2}{L} = \int_0^L \left( \frac{2F}{L} \sum_{j=1}^{\infty} \cos j\pi L \cos j \frac{\pi x}{L} \right) \left( 1 - \cos 2\pi x \right) dx
\]

which reduces to:

\[
\ddot{y}_i + \frac{y_i f^2}{L} = \frac{2F}{3mL} \left( 1 - \cos 2\pi t \right)
\]

because the integral of the product terms: \( \cos \frac{\pi x}{L} \cos j \frac{\pi x}{L} \)
is zero unless \( i = 2j \) which represents the orthogonality of the modes.

The complete solution including the natural response with the initial conditions of \( y = \dot{y} = 0 \) at \( t = 0 \) is:

\[
y = \frac{FL^3}{8\pi^2 EI} \sum_{i=1}^{\infty} \left( 1 - \cos 2i\pi \frac{x}{L} \right) \left[ \frac{1}{k^4} - \left( \cos i\pi t + \frac{n^2}{L^2 f^2} \cos 2\pi f t \right) \right]
\]

\[\text{eqn (14)}\]

As in the simply supported case the maximum response occurs when the frequency of the travelling load \( n = jf \). However the frequency \( n \) was defined as \( n = \frac{V}{L} \) whereas for the simply supported case \( \omega = \frac{V}{2L} \) hence comparison of responses for two beams with the different end conditions should reveal maxima at different \( \frac{V}{fL} \).

As in the case of the simply supported beam, the free vibration response, which results when the load reaches the end of the beam, is found by matching the displacement and velocity with that from the forced vibration. Thus:

\[
y = \frac{FL^3}{8\pi^2 EI} \sum_{i=1}^{\infty} \left( 1 - \cos 2i\pi \frac{x}{L} \right) \left[ \frac{n^2}{2f^2} \left( -\sin i\pi L \frac{f}{n} \sin 2\pi ft + (1 - \cos \frac{\pi f}{n}) \cos 2\pi ft \right) \right] \]

\[\text{eqn (14)}\]
3.7.3 Modification to approximate solution for the built-in case

The approximate mode shape was used in order to obtain a simple expression for the response function $Y_i$. However, having derived these expressions the exact mode shape may be substituted in the complete solution in order to obtain the correct distribution of bending moment. An associated modification of $2/1.588$ would also be required to account for differences in the unit deflections given by the two mode shapes. It was shown that the approximate method gave a good estimate of the fundamental frequency so the response function for the fundamental mode, which is in turn a function only of the natural and forcing frequencies, should give a good estimate of the response. In higher modes the fundamental frequencies are clearly in error and although the exact values could be substituted into the response function the result could still be in error because the resonance condition was found to depend on the mode shape in rather than the frequencies of the higher modes.

The full solution for the bending moment response is obtained as before using eqn (2). It can be shown that for a slowly moving load at the centre of the beam the bending moments compared with those from statics are:

<table>
<thead>
<tr>
<th></th>
<th>by dynamics</th>
<th>by statics</th>
</tr>
</thead>
<tbody>
<tr>
<td>at the ends</td>
<td>$M = 0.1446$ FL</td>
<td>$M = 0.125$ FL</td>
</tr>
<tr>
<td>at the centre</td>
<td>$M = -0.0873$ FL</td>
<td>$M = -0.125$ FL</td>
</tr>
<tr>
<td>range</td>
<td>$M = 0.2324$ FL</td>
<td>$M = 0.250$ FL</td>
</tr>
</tbody>
</table>

The dynamic magnification factor presented in Figs 1-3 have been based on a comparison of bending moment range since this eliminates the effects of end fixity on the static response.
3.8 Dynamic Magnification Factor

For the purposes of this report the dynamic magnification factor $K_d$ is defined as the ratio of peak dynamic stress in a given mode to the corresponding static stress caused by a uniformly distributed peak slam load.

In the case of a single degree of freedom lumped mass analogue the stress in the spring and the deflection are directly related and the dynamic magnification factor depends only on the natural frequency and the inclined slam load history. However the response of a beam is more complex and the dynamic magnification factor for stress is different to that for deflection and further depends on the end restraint of the beam.

The final step in obtaining a solution for the bending moment response to a slam load involves summing the responses to a series of point loads which present the distributed slam load. Due account must be taken of the phase differences of the individual responses as well as ensuring that the free vibration response is used after each load leaves the end of the beam.

A computer program was written in BASIC for the final part of the analysis and the listings are given in appendix 1, and the results of the calculations are given in figs 1, 2 and 3 together with experimental data.
4. RE-ANALYSIS OF EXPERIMENTAL DATA

4.1 Method of Re-Analysis

Data from slamming of a model structure of strain distributions and dynamic magnification factors was given in report OT-R-8042. The dynamic magnification factors were found to differ from those predicted by the lumped mass analogue by up to 28% but there was some uncertainty in the results due to the measured strains being less than those predicted from a static calibration. A constant calibration factor was applied to the strain measurements at all positions along the tubular beam; however this left an inconsistency between the predictions of end fixity from strain distribution and from measured frequency. Furthermore the measured strain distributions did not match those from the theories of statics or dynamics.

Subsequent to publishing the report on the slamming experiments, reports became available from the "UK Offshore Research Project". In particular those by Clayton et al (1978) and Fessler and Stanley (1977) contained a study of stress concentrations around tubular T-joints and it was clear that a large reduction in stress would be expected at those strain gauge sites on the model structures used in the slamming experiments which were close to the tube ends.

Therefore the experimental data has been re-analysed by applying a correction factor to the measured strains at the tube ends to give the bulk stresses appropriate to those from bending theory. The factor was determined from the static calibration assuming similar fixities applied at each end of the tube, and that the measured strains at the centre of the tube were the true bulk strains, i.e. no stress concentrations at the tube centre.

The dynamic magnification factors were abstracted by comparing the experimental mean strain range between the centre and ends of the tube with that given by simple bending theory using a static loading associated with the peak slam coefficient of 5.15. This method reduces the sensitivity of the results to end fixity since the static bending moment range is
independent of fixity. The effect of fixity is thus confined to the
dynamic magnification factors and the results are given in Figs 1, 2 and
3 together with the predictions from the theories of beam vibration
described earlier. The definition of the dynamic magnification factor
has similarities to that used by Ridley (1982) but is different to that
used in the original analysis by the ratio of the inclined impact peak
slam coefficient $C_{s0}$ to the horizontal peak slam coefficient $C_{s0}$.
Unfortunately this definition of the dynamic magnification factor masks
the degree to which the response contains free vibrations.

The results for the measured strain distributions, expressed as
a bending moment coefficient $K_m$, are given in Figs 4 and 5.

4.2 Discussion of Results

The re-analysed strain distributions are a much better match to the
theoretical distributions obtained from statics and dynamics, which are
similar. The end fixities obtained from the re-analysed strain
distributions are also much closer to those obtained from the frequency
analysis than was the case with the original analysis. These improvements
in the results together with the evidence of stress concentration causing
the reduced strain range measured during static calibrations leads to
greater confidence in the accuracy of the results obtained from the model
structure tests. A study of the experimental technique indicated that
the accuracy of the results should be within ±5%.

The dynamic magnification factors obtained from the first fixity
model (fixity $\geq 0.93$ of built-in ends) show similar trends to those
predicted by the beam vibration theories presented in this report although
the values are slightly closer to the results for a simply supported
than a built-in beam. However in either case the match is much closer
than with the lumped mass theory.

In the case of the second fixity model (fixity $\leq 0.87$) the
experimental data is still 15-20% higher than predicted although comparison
of the measured strain response with those predicted shows a much closer
match has been obtained with the beam vibration theories than the lumped
mass theory.
5. CONCLUSIONS

The solution for the dynamic response of a beam to an inclined slam using the Euler Bernoulli theory has resulted in a better match with experimental data and significant differences with the predicted response of a simple lumped mass system.

However, some experimental results for dynamic magnification factors are still greater than theory predicts and by more than the likely experimental error. So it is possible that the slam and response have not yet been perfectly modelled.
REFERENCES


<table>
<thead>
<tr>
<th>Investigator</th>
<th>Test type</th>
<th>Analysed data</th>
<th>Appropriate theories</th>
<th>£/D</th>
<th>0/D/V</th>
<th>N_F</th>
<th>£_SO</th>
</tr>
</thead>
<tbody>
<tr>
<td>Arhan &amp; Deleuil 1978</td>
<td>Drop 2-D</td>
<td>✓</td>
<td>Beam response by lumped mass.</td>
<td>3.3 to 10</td>
<td>-</td>
<td>1.6 to 2.8</td>
<td>2.4 to 6.9</td>
</tr>
<tr>
<td>Campbell &amp; Weynberg 1980</td>
<td>Drop 2-D &amp; 3-D</td>
<td>✓ ✓ ✓ ✓</td>
<td>Hydroelastic damping &amp; mass terms from Wagner approach strip theory for 3D load from 2D data, response by lumped mass analogue.</td>
<td>6 to 16</td>
<td>37 to 165</td>
<td>1.9 to 5.6</td>
<td>4.5 to 6.0</td>
</tr>
<tr>
<td>Hagiwara &amp; Yuhara 1974</td>
<td>Drop 2-D &amp; 3-D</td>
<td>✓ ✓</td>
<td>Von Karman. Response by lumped mass analogue.</td>
<td>0.19 to 0.63</td>
<td>0.44 to 1.8</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>Sarpkaya 1978</td>
<td>U-tube rising surface</td>
<td>✓ ✓</td>
<td>Wagner modified for cylinders, 3D load &amp; horizontal speed.</td>
<td>4.5 to 12</td>
<td>16 to 59</td>
<td>0.4 to 1.3</td>
<td>±0.05</td>
</tr>
<tr>
<td>Schnitzer &amp; Hathaway 1953</td>
<td>Drop 2-D</td>
<td>✓ ✓</td>
<td>Fabula &amp; Ruggles modified using slender body theory. Beam response using Fourier end plate series for deflection.</td>
<td>1.1 to 1.5</td>
<td>64 to 158</td>
<td>0.5 to 1.0</td>
<td>4.1 to 6.4</td>
</tr>
<tr>
<td>Sollie 1976 &amp; Faltings et al. 1977</td>
<td>Drop 2-D</td>
<td>✓ ✓</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 1. (Continued) Summary of experimental investigation of cylinder slamming (After Miller).

<table>
<thead>
<tr>
<th>Investigator</th>
<th>Test type</th>
<th>Analysed data</th>
<th>Appropriate theories</th>
<th>L/D</th>
<th>fD/V</th>
<th>N_F</th>
<th>C_{so}</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Peak Pressure Peak Load pressure distrib- load history</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Watanabe 1934</td>
<td>Drop 2-D</td>
<td>✓</td>
<td>✓</td>
<td>0.5</td>
<td>25</td>
<td>1.4 to 2.2</td>
<td>6.2</td>
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<tr>
<td>Canham 1977</td>
<td>Regular waves in a tank.</td>
<td>✓</td>
<td></td>
<td>15</td>
<td>12 to 68</td>
<td>0.4 to 0.6</td>
<td>No slam observed.</td>
</tr>
<tr>
<td>Dalton 1976</td>
<td>Regular waves in a tank</td>
<td>✓</td>
<td></td>
<td>20</td>
<td>5 to 24</td>
<td>0.5</td>
<td>1 to 4.5</td>
</tr>
<tr>
<td>Holmes et al. 1976</td>
<td>Regular waves in a tank</td>
<td>✓</td>
<td></td>
<td>24 to 96</td>
<td>3 to 55</td>
<td>0.3 to 3.4</td>
<td>0.4 to 2.9</td>
</tr>
<tr>
<td>Miller 1977</td>
<td>Irregular wave train in a tank</td>
<td>✓</td>
<td>Response by lumped mass analogue. Strip theory for 3-D load from 2-D Von Karman theory.</td>
<td>15</td>
<td>19 to 94</td>
<td>0.3 to 1.5</td>
<td>1 to 8</td>
</tr>
<tr>
<td>Webb 1977</td>
<td>Irregular waves full scale</td>
<td>✓</td>
<td></td>
<td>34</td>
<td></td>
<td></td>
<td>3 to 4</td>
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</table>

Notes 1. The determination of the slam velocity U differs between the experiments in waves.
<table>
<thead>
<tr>
<th>Model</th>
<th>Measured frequency, Hz</th>
<th>Average end fixity ratio to encastré beam with uniform load.</th>
<th></th>
</tr>
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<tbody>
<tr>
<td></td>
<td>fundamental</td>
<td>higher</td>
<td>ringing</td>
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<tr>
<td>1st fixity</td>
<td>220</td>
<td>1200 (h)</td>
<td>3200</td>
</tr>
<tr>
<td></td>
<td></td>
<td>620 (i)</td>
<td></td>
</tr>
<tr>
<td>2nd Fixity</td>
<td>180</td>
<td>1100 (h)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>560 (i)</td>
<td></td>
</tr>
</tbody>
</table>
FIGURE 1

DYNAMIC MAGNIFICATION FACTORS FOR 1st FIXITY MODEL.

1st fixity model \( \frac{V}{fD} = 0.235 \)

Dynamic magnification factor \( K_d = \frac{\text{Peak strain}}{\text{Static strain at peak load}} \)
FIGURE 2
DYNAMIC MAGNIFICATION FACTORS FOR 2nd FIXITY MODEL.

2nd fixity model.  \( \frac{V}{fD} = 0.287 \)
FIGURE 3

DYNAMIC MAGNIFICATION FACTORS FOR HORIZONTAL IMPACTS.

Experimental results

1st fixity
2nd fixity

Beam vibration theory
Simply supported ends
Built in ends

Dynamic magnification factor $k_d = \frac{\text{Peak strain}}{\text{Static strain @ peak load}}$

$V/\bar{E}D$
FIGURE 4

STATIC BENDING MOMENT DISTRIBUTIONS FROM STRAIN MEASUREMENTS ON A MODEL STRUCTURE AT VARIOUS INCLINATIONS.

1st fixity, fundamental mode

Experimental Results
- 30°
- 0° av. at 3 values $N_f$
- 1° peak at $Y/L = 0.50$

simple theory for uniform load and fixity = 0.9
FIGURE 5

STATIC BENDING MOMENT DISTRIBUTIONS FROM STRAIN MEASUREMENTS ON A MODEL STRUCTURE AT VARIOUS INCLINATIONS.

2nd fixity, fundamental mode

Bending moment coefficient

Experimental Results

- $\theta = 40'$
- $= 32'$
- $= 24'$
- $= 16'$
- $= 8'$

Simple theory for uniform load and fixity $= 0.85$
APPENDIX I

COMPUTER LISTINGS
SIMPLY SUPPORTED BEAM RESPONSE

FILE NO: 2

3/8/81

100 REM BEAM RESPONSE TO INCLINED SLAM IMPACT

110 REM INPUT DATA
120 INIT
140 PRINT "ENTER SLAM VELOCITY FT/S "
150 INPUT V
160 PRINT "ENTER IMPACT ANGLE DEG "
170 INPUT G
180 Q=Q*PI/180
190 PRINT "ENTER CYLINDER LENGTH FT "
200 INPUT L
210 PRINT "ENTER CYLINDER DIAMETER FT "
220 INPUT D
230 PRINT "ENTER NO. OF TIME STEPS FOR SPRAY ROOT TO CROSS BEAM"
240 INPUT I1

250 REM CALCULATE NATURAL FREQUENCY (RAD/S)
260 N1=PI^2/L^2*D*(3.0E+7/8/0.283/12*32.2)^0.5
265 END

270 REM CALCULATE RISE FREQUENCY (RAD/S)
280 N=V/TAN(Q)/2/L^2*PI

290 REM CALCULATE TIME STEP
300 T1=L*TAN(Q)/V
310 T2=L*TAN(Q)/V
315 PRINT USING 320:"NATURAL FREQUENCY = "TAN/(D) " Hz"
316 PRINT USING 320:"RISE FREQUENCY = "TAN/(D) " Hz"
320 PRINT USING 430:V/(N1*D)="",V/(N1/2/P1*D),"L*TAN(D)=",L*TAN(Q)/D
340 PRINT 3."SIMPLY SUPPORTED BEAM RESPONSE TO INCLINED SLAM U,J"
347 PRINT "H",L="",F",P1=""D="",G="",Q="",V="",TAN="",RAD"
348 PRINT 3."N U,J="",L="",F",P1=""D="",G="",Q="",V="",TAN="",RAD"
349 PRINT 3."N U,J="",L="",F",P1=""D="",G="",Q="",V="",TAN="",RAD"
344 PRINT 3."N U,J="",L="",F",P1=""D="",G="",Q="",V="",TAN="",RAD"
345 PRINT 3."N U,J="",L="",F",P1=""D="",G="",Q="",V="",TAN="",RAD"
346 PRINT 3."N U,J="",L="",F",P1=""D="",G="",Q="",V="",TAN="",RAD"
347 PRINT 3."N U,J="",L="",F",P1=""D="",G="",Q="",V="",TAN="",RAD"
350 REM CALCULATE MUSCROSS BEAM
360 REM IN FREQUENCY
370 IF M<0.01 THEN 380
380 T=0.1
390 PRINT 24, USING 347:11,"TIME STEPS TO CROSS BEAM 
395 PRINT 34, USING 347:12,"TIME STEPS IN TOTAL  
400 DIM C(I2+1),W(I2),M(I2,3),M1(I2,3),T(I2)
410 W1=5.15*L
415 T=0
420 C=0
430 W=0
440 M=0
450 M1=0

460 REM FOR TIME STEP NO. I
470 FOR I=1 TO I2
480 T(I)=I*T1

490 REM CALCULATE POINT LOAD FROM INTEGRATION OF LOAD DISTRIBUTION
494 GO TO 497
495 C(I+1)=5.15*L/I1*I
496 GO TO 520
497 IF V*T(I)/D(1,2) THEN 500
498 W(I)=I*L/I1
499 GO TO 530
500 C(I+1)=5.15/19*D/TAN(D)*LOG(1+19*V*T(I)/D)
510 C(I+1)=C(I+1)+0.55/2*V^2*T(I)^2/D/TAN(D)
520 W(I)=C(I+1)-C(I)

530 REM FOR LOAD NO. H

540 REM AT ITH TIME STEP THE ARE I LOADS ON THE BEAM
550 FOR H=0 TO I-1
600 REM FOR EACH MODE SHAPE J
610 FOR J=1 TO 3
620 M(H+1,J)=16/PI^2*W(H+1)/W1/(J^2-N^2/N1^2)

640 REM IF LOADS HAVE REACHED THE END OF THE BEAM VIBRATION IS FREE  
645 IF T(I-H)<T2 THEN 650
646 M2=(COS(J*PI)*COS(J^2*N1*T2)-1)*SIN(J^2*N1*T(I-H))  
647 M3=COS(J*PI)*SIN(J^2*N1*T2)*COS(J^2*N1*T(I-H))  
648 GO TO 660
650 M(H+1,J)=M(H+1,J)*(SIN(J*N*T(I-H))-N/(J*N1)*SIN(J^2*N1*T(I-H)))
REM SUM THE RESPONSES FROM EACH LOAD
M1(I,J)=M1(I,J)+M(H+1,J)
NEXT J
NEXT I
VIEWPORT 10,130,0,50
WINDOW 0,T(I2-1),-0.5,2
AXIS T1,0.1,0,0
AXIS 0.0,T(I1),1
MOVE T(I1),-0.5
PRINT USING 738:"Vtr/D=",V*T(I1)/D,"TIC INTERVAL="",V*T1/D
IMAGE13A,2D,2D,4X,13A,2D,3D,"L_"
MOVE 0.1
PRINT "1.0"
MOVE 0.2
PRINT "DYNAMICJ_MAGNIFICATIONJ_FACTOR"
MOVE 0,0
FOR J=1 TO 3
FOR I=1 TO I2
DRAW T(I),M1(I,J)
NEXT I
MOVE 0,0
NEXT J
GO TO 2000
FOR I=1 TO I2-1 STEP 2
DRAW T(I),M5(I)
MOVE T(I+1),M5(I+1)
NEXT I
MOVE 0,0
PRINT @4:"DYNAMIC MAGNIFICATION FACTORS FOR BENDING MOMENTS"
PRINT @4:"NORMALISED BY L=2/7"
PRINT @4:"V*T/D MODE 1 MODE 2 MODE 3 MODES 1-3 ANALOGUE"
FOR I=1 TO I2-1
M4=M1(I,1)-M1(I,3)
CALL "WAIT",0.5
PRINT @4:USING 1030:T(I)*V/D,M1(I,1),M1(I,2),M1(I,3),M4,M5(I)
NEXT I
PRINT @4:"L_"
END

REM DYNAMIC MAGNIFICATION FACTOR FROM SIMPLE ANALOGUE
T3=T1/100
DELETE C1,M5
DIM C1(I2),M5(I2)
C1=0
M5=0
V1=L/D*TAN(Q)
FOR J=1 TO I2
FOR K=1 TO 100
V2=V*13/D*(100*(J-1)+K)
IF V2>V1 THEN 2080
C1(J)=5.15/(19*V1)*LOG((1+19*V2)+0.55*V2*2/2/V1)
GO TO 2090
C1(J)=5.15/(19*V1)*LOG((1+19*V2)/(1+19*(V2-V1)))+0.55*(V2-V1/2)
IF J=1 THEN 2150
IF K=1 THEN 2150
X=0
X2=C1(1)
GO TO 2170
X2=C1(J)-N1*X
X=X+X1*T3+X2*T3*2/2
X1=X1+X2*T3
NEXT K
M5(J)=X*N1*2/5.15
NEXT J
3000 REM TO PLOT OUT THE RESPONSES
3010 VIEWPORT 10,130,0,70
3020 WINDOW 0,T(I2-1),-0.5,2
3030 AXIS @S:T(1),0,1,0
3040 AXIS @S:0,0,T(I1),1
3050 MOVE @S:T(I1),-0.5
3060 PRINT @S: USING 3070;"VT Tr/D=";V*T(I1)/D;"TIC INTERVAL=";V*T1/D
3070 IMAGE"",6A;2D;2D;4X;13A;2D;5D;"K="
3080 MOVE @S:0,1
3090 PRINT @S:"1.0"
3100 MOVE @S:0,1.9
3110 PRINT @S: USING 3120;"DYNAMIC","MAGNIFICATION","FACTOR"
3120 IMAGE"",7A;"J_","7("",13A;"J_",13("",6A
3130 MOVE @S:0,0
3140 FOR J=1 TO 3
3150 FOR I=1 TO I2
3160 DRAW @S:T(I),M1(I,J)
3170 NEXT I
3180 MOVE @S:0,0
3190 NEXT J
3200 MOVE @S:0,0
3210 FOR I=1 TO I2-1 STEP 2
3220 DRAW @S:T(I),M5(I)
3230 MOVE @S:T(I+1),M5(I+1)
3240 NEXT I
3250 VIEWPORT 0,150,80,100
3260 WINDOW 0,150,80,100
3265 MOVE @S:0.97
3270 PRINT @S:" SIMPLY SUPPORTED BEAM RESPONSE TO INCLINED SLAM IMPACT"
3280 PRINT @S:
3290 PRI @S: USI 3300;"L="",L,"FT","D="",D,"FT","V="",V,"FT/S","Q="",Q,"RAD
3300 IMAGE2(2A,3D,2D,X,2A,3X),2A,2D,2D,X,4A,3X,2A,2D,3D,X,3A
3310 PRINT @S:
3320 PRINT @S: USING 3330;"NATURAL FREQUENCY="",N1/2/PI,"HZ"
3330 IMAGE20A,4D,D,X,2A
3340 PRINT @S: USING 3330;"RISE FREQUENCY="",N/2/PI,"HZ"
3350 END
BUILT IN BEAM RESPONSE

FILE NO: 3

3/8/81

100 REM BUILT IN BEAM RESPONSE TO INCLINED SLAM IMPACT

110 REM INPUT DATA
120 INIT
140 PRINT "ENTER SLAM VELOCITY FT/S"
150 INPUT V
160 PRINT "ENTER IMPACT ANGLE DEG"
170 INPUT Q
180 Q=Q*PI/180
190 PRINT "ENTER CYLINDER LENGTH FT"
200 INPUT L
210 PRINT "ENTER CYLINDER DIAMETER FT"
220 INPUT D
230 PRINT "ENTER NO. OF TIME STEPS FOR SPRAY ROOT TO CROSS BEAM"
240 INPUT N

250 REM CALCULATE NATURAL FREQUENCY (RAD/S)
260 N1=PI*2/L*2*D*(3.0E+7/8/0.283/12*32/2)*0.5*(16/3)*0.5
265 END

270 REM CALCULATE RISE FREQUENCY (RAD/S)
280 N=V/TAN(Q)/L*2*PI

290 REM CALCULATE TIME STEP
300 T1=L*TAN(Q)/V/11
305 T2=L*TAN(Q)/V
310 PRINT USING 320: "NATURAL FREQUENCY = ",N1/2/PI," HZ"
315 PRINT USING 320: "RISE FREQUENCY = ",N/2/PI," HZ"
320 IMAGE 120A,4D,10,3A
330 PRINT USING 340: VO/(N1*D)="V/(N1/2/PI*D),LTANG/D="L*TAN(Q)/D
340 IMAGE 9A,3D,3D,4X,8A,3D,3D
341 PRINT @4:"BUILT IN BEAM RESPONSE TO INCLINED SLAM J..."
350 REM CALCULATE BENDING MOMENT RESPONSE
360 I2=INT(2*PI/N1/T1)
370 IF I2<2*I1 THEN 390
380 GO TO 392
390 I2=2*I1
392 PRINT @4: USING 347;I1,"TIME STEPS TO CROSS BEAM"
393 PRINT @4: USING 347;I2,"TIME STEPS IN TOTAL"
394 DELETE C,W,M,M1,T
395 DIM C(I2+1),W(I2),M(I2,1),M1(I2,1),T(I2)
400 W1=5.15*L
410 T=0
420 C=0
430 W=0
440 M=0
450 M1=0

460 REM FOR TIME STEP NO. I
465 PRINT @4: "RESPONSE FROM EACH LOAD M(H+1,1) FOR H=0 TO 9 @ TIME T(I)"
470 FOR I=1 TO I2
480 T(I)=I*T1

490 REM CALCULATE POINT LOAD FROM INTEGRATION OF LOAD DISTRIBUTION
494 GO TO 497
495 C(I+1)=5.15*L/I1*I
496 GO TO 520
497 IF V*T(I)/D(1,2) THEN 500
498 W(I)=1*L/I1
499 GO TO 530
500 C(I+1)=5.15/19*D/TAN(Q)*LOG(1+19*V*T(I)/D)
510 C(I+1)=C(I+1)+0.35/2*V*T(I)^2/D/TAN(Q)
520 W(I)=C(I+1)-C(I)

530 REM FOR LOAD NO. H

540 REM AT ITH TIME STEP THE ARE I LOADS ON THE BEAM
550 FOR H=0 TO I-1

620 REM FOR EACH MODE SHAPE J
630 FOR J=1 TO 1
640 M(H+1,J)=8/PI^2*W(H+1)/W1*0.232/C,202

642 REM IF LOADS HAVE REACHED THE END OF THE BEAM VIBRATION IS FREE
643 IF T(I-H)(T2 THEN 650
645 M2=SIN(2*PI*N1/N)*SIN(N1*T(I-H))
646 M3=(COS(2*PI*N1/N)-1)*COS(N1*T(I-H))
647 M(H+1,J)=M(H+1,J)*N^2/N1^2/(1-N^2/N1^2)*(M2+M3)*-1
648 GO TO 660
650 MB=1-(COS(N*T(I-H))-N^2/N1^2*COS(N1*T(I-H)))/(1-N^2/N1^2)
651 M(H+1,J)=M(H+1,J)*MB
660 REM SUM THE RESPONSES FROM EACH LOAD
670 M1(I,J)=M1(I,J)+M(H+1,J)
680 NEXT J
690 NEXT H
691 GO TO 700
695 PRINT @4: USING 697:M(1,1),M(2,1),M(3,1),M(4,1),M(5,1):
696 PRINT @4: USING 698:M(6,1),M(7,1),M(8,1),M(9,1),M(10,1)
697 IMAGE 5( 2D,3D, X),S
698 IMAGE 5( 2D,3D, X)
700 NEXT I
710 VIEWPORT 10,130,0,50
720 WINDOW 0,T(I2-1),-0.5,2
730 AXIS T1,0.1,0,0
735 AXIS 0,0,T(I1),1
736 MOVE T(I1),-0.5
737 PRINT USING 738:"VTR/D= "",V*T(I1)/D," TIC INTERVAL=",V*T1/D
738 IMAGE13A,2D,2D,4X,13A,2D,3D,"L_"
739 MOVE 0,1
740 PRINT "1.0"
741 MOVE 0,2
742 PRINT " DYNAMICJ_MAGNIFICATIONJ_FACTOR"
745 MOVE 0,0
749 FOR J=1 TO 1
750 FOR I=1 TO I2
760 DRAW T(I),M1(I,J)
770 NEXT I
780 MOVE 0,0
790 NEXT J
800 GO TO 2000
805 FOR I=1 TO I2 STEP 2
810 DRAW T(I),M5(I)
815 MOVE T(I+1),M5(I+1)
820 NEXT I
830 MOVE 0,0
970 PRINT @4:"DYNAMIC MAGNIFICATION FACTORS FOR BENDING MOMENTS"
975 PRINT @4:"NORMALISED BY Cso*L'2/8"
980 PRINT @4:"V*T/D MODE 1 MODE 2 MODE 3 MODES 1-3 ANALOGUE"
1000 FOR I=1 TO I2-1
1015 CALL "WAIT",0.5
1020 PRINT @4: USING 1030:T(I)*V/D,M1(I,1),M5(I)
1030 IMAGE2(2D,2D,4X),27X,2D,2D
1040 NEXT I
1045 PRINT @4:"L_"
1050 END

1990 REM DYNAMIC MAGNIFICATION FACTOR FROM SIMPLE ANALOGUE
2000 T3=T3/100
2003 DELETE C1,M5
2005 DIM C1(I2),M5(I2)
2006 C1=0
2007 M5=0
2010 V1=L/D*TAN(Q)
2020 FOR J=1 TO I2
2030 FOR K=1 TO 100
2040 V2=V*T3/N*(100*(J-1)+K)
2050 IF V2<0 THEN 2080
2060 C1(J)=5.15/(19*V1)*LOG(1+19*V2)+0.55*V2*2/2/V1
2070 GO TO 2090
2080 C1(J)=5.15/(19*V1)*LOG((1+19*V2)/(1+19*(V2-V1)))+0.55*(V2-V1/2)
2090 IF J>1 THEN 2150
2100 IF K<1 THEN 2150
2110 X=0
2120 X1=0
2130 X2=C1(I)
2140 GO TO 2170
2150 X2=C1(J)-N1^2*X
2160 X=X+X1*T3+X2*T3^2/2
2170 X1=X1+X2*T3
2180 NEXT K
2190 M5(J)=X*N1^2/5.15
2200 NEXT J
2210 GO TO 805

3000 REM TO PLOT OUT THE RESPONSES
3010 VIEWPORT 10,130,0,70
3020 WINDOW 0,T(I2-1),-0,5,2
3030 AXIS @5:T1,0,1,0,0
3040 AXIS @5:0,0,T(I1)-1,1
3050 MOVE @5:T(I1),-0.5
3060 PRINT @5: USING 3070;"VT/D=";V*T(I1)/D,"TIC INTERVAL=";V*T1/D
3070 IMAGE",6A,2D,2D,4X,13A,2D,3D,"K_"
3080 MOVE @5:0,1
3090 PRINT @5;"1.0"
3100 MOVE @5:0,1.9
3110 PRINT @5: USING 3120;"DYNAMIC","MAGNIFICATION","FACTOR"
3120 IMAGE4*X,7A,"J_",7(""),13A,"J_",13(""),6A
3130 MOVE @5:0,0
3140 FOR J=1 TO 1
3150 FOR I=1 TO 12
3160 DRAW @5:T(I),M1(I,J)
3170 NEXT I
3180 MOVE @5:0,0
3190 NEXT J
3200 MOVE @5:0,0
3210 FOR I=1 TO I2-1 STEP 2
3220 DRAW @5:T(I),M5(I)
3230 MOVE @5:T(I+1),M5(I+1)
3240 NEXT I
3250 VIEWPORT 0,150,80,100
3260 WINDOW 0,150,80,100
3265 MOVE @5:0.97
3270 PRINT @5;" BUILT IN BEAM RESPONSE TO INCLINED SLAM IMPACT"
3280 PRINT @5:
3290 PRI @5: USI 3300;"L=";L,"FT","D=";D,"FT","V=";V,"FT/S","D=";Q,"RAD"
3300 IMAGE2(2A,3D,2D,X,2A,3X),2A,2I,2D,2D,X,4A,3X,2A,2D,3D,X,3A
3310 PRINT @5:
3320 PRINT @5: USING 3330;"NATURAL FREQUENCY=";N1/2/PI,"HZ"
3330 IMAGE20A,4D,D,X,2A
3340 PRINT @5: USING 3230; "RISE FREQUENCY=";N/2/PI,"HZ"
3350 END